ECS 452: Additional Example

1. Consider a BSC whose crossover probability is p = 0.2. A channel code uses the following codebook:

s	**x**
0	011
1	100

$\underline{x} = 011$

   a. Suppose the codeword 011 was transmitted.
   What is the probability that the channel output is 101? $\underline{y} = 101$

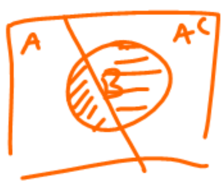
$P[\underline{Y} = 101 \mid \underline{X} = 011] = Q(101 \mid 011) = 0.2 \times 0.2 \times 0.8 = 0.032$

   b. Assume that the two possibilities of the info-bit S are equally likely. Suppose we observed 101 at the output of this channel.

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

      i. What is the probability that the codeword 011 was transmitted?

$P[\underline{X} = 011 \mid \underline{Y} = 101] = P(A|B) \equiv \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$



$= \frac{P[\underline{Y} = 101 \mid \underline{X} = 011]\, P[\underline{X} = 011]}{P[\underline{Y} = 101]}$ (0.032, 0.5)

$P[\underline{Y} = 011] = P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$

      ii. What is the probability that the codeword 100 was transmitted?

$P[\underline{X} = 100 \mid \underline{Y} = 101]$

      iii. At the receiver, if a MAP decoder is used, find the decoded codeword ($\hat{\underline{x}}$) and the corresponding decoded info-bit ($\hat{s}$).

$\underline{x}$	$d(\underline{x}, 101)$
011	2
100	1 ← min

$\hat{\underline{x}}_{MAP}(101) = 100$

$\hat{s}_{MAP}(101) = 1$

crossover prob. (BSC, $p < 0.5$)

MAP $\xrightarrow{\text{Equally-likely input}}$ ML $\xrightarrow{\text{BSC } p<0.5}$ min. distance decoding ~~$\xrightarrow{\text{repetition code}}$~~ majority voting